DEPARTMENT OF MATHEMATICS TELANGANA UNIVERSITY, NIZAMABAD - 503 322

M.Sc. (MATHEMATICS) COURSE STRUCTURE (with effect from 2014– 2015)

SEMESTER I

Paper	Paper Code	Name of the Paper	Instruction Hrs/Week	Duration of Exam (in Hrs)	Marks		Total		
					UE	IA	Marks	Credits	
	Theory Papers								
I	MM101	Algebra	6	3	70	30	100	6	
II	MM102	Real Analysis	6	3	70	30	100	6	
III	MM103	Topology	6	3	70	30	100	6	
IV	MM104	Elementary Number Theory	6	3	70	30	100	6	
V	MM105	Mathematical Methods	6	3	70	30	100	6	
Seminar		2							
Total			30+2		350	150	500	30	

SEMESTER II

Paper	Paper Code	Name of the Paper	Instruction Hrs/Week	Duration of Exam (in Hrs)	Marks		Total		
					UE	IA	Marks	Credits	
	Theory Papers								
I	MM201	Advanced Algebra	6	3	70	30	100	6	
II	MM202	Advanced Real Analysis	6	3	70	30	100	6	
III	MM203	Functional Analysis	6	3	70	30	100	6	
IV	MM204	Theory Of Ordinary Differential Equations	6	3	70	30	100	6	
V	MM205	Discrete Mathematics	6	3	70	30	100	6	
Seminar		2							
Total			30+2		350	150	500	30	

DEPARTMENT OF MATHEMATICS TELANGANA UNIVERSITY, NIZAMABAD – 503 322

M.Sc. (MATHEMATICS) COURSE STRUCTURE

(with effect from 2015– 2016)

SEMESTER III

D	Paper Code	Name of the Paner	Instruction Hrs/Week	Duration of Exam (in Hrs)	Marks		Total		
Paper					UE	IA	Marks	Credits	
	Theory Papers								
I	MM301	Complex Analysis	6	3	70	30	100	6	
II	MM302	Elementary Operator Theory	6	3	70	30	100	6	
III		ELECTIVE	6	3	70	30	100	6	
	MM303 A	Mechanics							
	MM303 B	Analytic Number Theory							
	MM303 C	Operations Research							
IV		ELECTIVE	6	3	70	30	100	6	
	MM304 A	Commutative Rings							
	MM304 B	Integral Equations							
	MM304 C	Coding Theory							
V		ELECTIVE	6	3	70	30	100	6	
	MM305 A	Algebraic Number Theory							
	MM305 B	Application of Functional Analysis							
	MM305 C	Numerical Techniques							
	Seminar		2						
	Total		30+2		350	150	500	30	

SEMESTER IV

		SEI'	WESTER IV					
Paper	Paper Code	Name of the Paper	Instruction Hrs/Week	Duration of Exam (in Hrs)	Marks		Total	
					UE	IA	Marks	Credits
		The	ory Papers					
I	MM401	Advanced Complex Analysis	6	3	70	30	100	6
II	MM402	General Measure Theory	6	3	70	30	100	6
III		ELECTIVE	6	3	70	30	100	6
	MM403 A	Object Oriented Programming Through C++	4					
	MM403 B	Prime Number Theory	6					
	MM403 C	Advanced O. R.	6					
IV		ELECTIVE	6	3	70	30	100	6
	MM404 A	Banach Algebra						
	MM404 B	Projective Planes						
	MM404 C	Fluid Mechanics						
V		ELECTIVE	6	3	70	30	100	6
	MM405 A	Calculus Of Variations						
	MM405 B	Number Theory and Cryptology						
	MM405 C	Dynamical Systems						
	MM451	Practical : OOPS through C++	2*				Grade‡	
		Seminar	2					
		Total	30+2		350	150	500	30

^{*} per batch

[‡] Grades: **A**-Excellent, **B**-Good, **C**-Satisfactory, **D**-Unsatisfactory

MM 101 Paper-I

Algebra

Unit I

- 1. Normal Subgroups.
- 2. Quotient groups. Isomorphism theorems,
- 3. Automorphisms.
- 4. Conjugacy and G-sets. Normal series solvable groups Nilpotent groups
- 5. Alternating group A_n Simplicity of A_n.

 (No question is to be set in section 1 & 2. Pages 91 to 137 of [1])

UNIT-II

Structure theorems of groups – Direct products – finitely generated abelian groups – Invariants of finite abelian group – Sylows theorem – Groups of order p², pq. (Pages 138 to 158)

UNIT-III

Ideals and homomorphisms. Sum and direct sum of ideals. Maximal and prime ideals. Nil potent and nil ideals. Zorn's Lemma, UFD, PID, Euclidean domains. Polynomial rings over UFD. (Pages 179 to 219 of [1])

UNIT-IV

Rings of fractions. Rings with Ore conditions. Pages 224 to 232 Modules and vector spaces. Definition examples. Submodules and direct sums. R-homomorphisms and quotient modules completely reducible modules – free modules – representation of linear mappings. (Pages 244 to 280)

Text Book: [1]. Basic Abstract Algebra by P.B. Bhattacharya, SK Jain, S R. Nagapaul

Reference Books: 1. Topics in Algebra, by I. N. Herstein 2. Basic Algebra, by N. Jacobson

MM102 Paper-II

Real Analysis

UNIT I

Series: Upper and lower limits, addition and multiplication of series, rearrangements, Continuity: Limits of continuous functions, continuity and compactness, continuity and connectedness, discontinuity, monotonic functions, infinity limits and limits at infinity

UNIT II

The Reimann Steiltjes integral, definition and existence of integral, properties of the integral, integration and differentiation, Rectifiable curves.

UNIT III

Sequences and series of functions, discussions of main problem, uniform convergence, uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, weistrass approximation theorem

UNIT IV

Functions of several variables, linear transformations, differentiation, the contraction principle.

Text Book:- Principles of Mathematical Analysis by Walter Rudin,McGraw Hill International Edition. (Scope as in chapters 3, 4, 6, 7, 9)

MM-103 Paper-III

TOPOLOGY

UNIT-I

Concepts of Metric Spaces-Topological spaces-Definition and some examples-Definition of continuous mapping-Open mapping, homomorphism-Elementary concepts-Neighbourhood of a point Derived set, closed set, closure of a set. Dense set. Isolated point-Interior of a set, boundary of a set-Kuratowski closure axiom- open bases and sub bases- second countable space-Lindelof's theorem. Equivalent form of continuous function (Section 16 to 18 of Chapter III).

UNIT-II

Compactness open cover and subcover – Finite intersection property-Equivalent form of compactness of a topological space- Heine-Borel theorem-Compactness for metric spaces- Sequential compactness. Bolzano-weierstrass property- total boundedness-Lebesgue's covering lemma and its consequence on a continuous function defined on a compact metric space- Equivalent from of compactness of metric space-Ascolli's theorem. (Section 21, 24, 25 Chapter IV).

UNIT-III

Seperation- T_1 spaces and Hausdorff spaces- Equivalent form a T_1 -space- Relation between Hausdorff space and a T_1 -space-complete regular space and Normal space-Urysohn's lemma – Relation between a complete regular space and Hausdorff space-Tietze extension theorem – The Urysohn Imbedding theorem. (Section 26,27, 28 and 29 of Chapter-IV)

UNIT-IV

Connectedness- definition –disconnectedness- connectedness on real line – property of a continuous function defined on a connected space- components of a space- main property of components- product space- product topology- Tychonoff's theorem. Generalized Heine-Borel theorem, Product of Hausdorff spaces is Hausdorff and Product of connected spaces is connected.

(Section 31, 32 of Chapter VI , Section 22 and 23 of Chapter –IV and Section 26 of Chapter-V , Section 31 0f Chapter VII)

Recommended Text:- G.F.Simmons, "Topology and Modern Analysis", McGraw Hill

MM 104 Paper-IV

Elementary Number Theory

UNIT-I

The Fundamental Theorem of arithmetic: Divisibility, GCD, Prime Numbers, Fundamental theorem of Arithemtic, the series of reciprocal of the Primes, The Euclidean Algorithm.

UNIT-II

Arithmetic function and Dirichlet Multiplication, The functions (n), (n) and a relation connecting them, Product formulae for (n), Dirichlet Product, Dirichlet inverse and Mobius inversion formula and Mangoldt function (n), multiplication function, multiplication function and Dirichlet multiplication, Inverse of a completely multiplication function, Liouville's function (n), the divisor function is (n)

UNIT-III

Congruences, Properties of congruences, Residue Classes and complete residue system, linear congruences conversion, reduced residue system and Euler Fermat theorem, polynomial congruence modulo P, Lagrange's theorem, Application of Lagrange's theorem, Chinese remainder theorem and its application, polynomial congruences with prime power moduli

IINIT-IV

Quadratic residue and quadratic reciprocity law, Quadratic residues, Legendre's symbol and its properties, evaluation of (-1/p) and (2/p), Gauss Lemma, the quadratic reciprocity law and its applications.

Text Book:- Introduction to analytic Number Theory by Tom N. Apostol. Chapters 1,2,5,9

MM-105 Paper-V

Mathematical Methods

UNIT -I

Existence and Uniqueness of solutions of first order ordinary differential equations – Sturm-Liouville's boundary problems – Green's Function - Cauchy Problems and characteristics-Lagrange's and Charpits methods of solving first order partial differential equations.

UNIT-II

Partial differential equation of higher order with constant coefficients - Classification of second order PDE, separation of variables - heat, wave and Laplace equations.

UNIT-III

Solutions of Differential Equations in Power series - Bessel, Legendre Equations – Bessel function and its properties - Recurrence and generating functions - Legendre function of first kind and its properties- Recurrence relations and generating functions - Orthogonality of Bessel, Legendre polynomials. Rodrigue's formula for Pn(x).

UNIT-IV

Hermite and Leguerre's equations (Their polynomials) – Their properties including Recurrence relations-Orthogonal Properties of $L_n(x)$ and $H_n(x)$ - generating functions for both the polynomials- Rodrigue's formula for $L_n(x)$ and $H_n(x)$.

Text Books:-

- 1). S. G. Deo, V. Lakshmi Kantham, V. Raghavendra, "Ordinary Differential Equations and Stability Theory".
- 2). I.N.Sneddon, "Elementary or Partial Differential Equations", Mc.Graw Hill, 1988. 3). R.V.Churchill, "Operational Mathematics", McGraw Hill.
- 4). Frank Ayres, "Theory and Problems of Differential Equations, Mc GrawHill.

Reference Book:-

1). M.D.Raisingania, "Ordinary and Partial Differential Equations", S.Chand Comp. 2). C. R. Mondal, "Textbook of Ordinary Differential Equations"

MM 201 Paper-I

Advanced Algebra

UNIT-I

Algebraic extension of fields

- 1. Irreducible polynomials and Eisenstein criterion,
- 2. adjunction of roots
- 3. algebraic extensions
- 4. algebraically closed fields (Pages 281 to 299 of [1])

UNIT-II

Normal and separable extensions

- 1. Splitting Fields
- 2. Normal Extensions
- 3. Multiple roots
- 4. Finite Fields
- 5. Separable Extensions (Pages 300 to 321 of [1])

UNIT-III

Galois Theory

- 6. Automorphism group and fixed fields
- 7. Fundamental theory of Galois Theory
- 8. Fundamental theorem of Algebra (Pages 322 to 339 of [1])

UNIT-IV

Applications of Galois Theory to Classical problems

- 1. Roots of Unity and Cyclotonic Polynomials
- 2. Cyclic Extensions
- 3. Polynomials solvable by radicals
- 4. Symmetric Functions
- 5. Ruler and Compass Construction.

(Pages 340 to 366 of [1])

Text Book:- [1].Basic Abstract Algebra , By P. B. Bhattacharya, SK Jain, SR Nagapaul

Reference Book:- 1). Topics in Algebra, by I. N. Herstein

- 2). Contemporary Abstract Algebra, by Joseph A. Galelian
- 3). Basic Algebra, by N. Jacobson.

MM 202 Paper-II

Advanced Real Analysis

UNIT-I

Algebra of sets, Lebesgue measure, Introduction, Outer measure, measurable set and Lebesgue measure, a non-measurable set, measurable function, Little wood three principles.

UNIT-II

The Riemann Integral, the Lebesgue integral of a bounded function over a set of finite measure, the integral of a non-negative function, the general Lebesgue Integral, convergence in measure.

UNIT-III

Differentiation of monotonic function, functions of bounded variation, differentiation of an integral, absolute continuity, the L -spaces, the Minkowski and Holder inequalities, convergence and completeness.

UNIT-IV

Functions of several variables, the inverse function theorem, the implicit function theorem, the rank theorem, determinants, derivatives of higher order, Taylor's theorem, differentiation of integrals.

Text Books:

- 1. Real Analysis by H. L. Royden Chapters 3,4,5
- 2. Principles of Mathematical Analysis by Walter Rudin Chapter 9

MM 203 Paper-III

Functional Analysis

UNIT-I

Normed Space, Banach Space, further properties of normed spaces, Finite dimensional normed spaces and subspaces, compactness and finite dimension linear operators, Bounded and continuous linear operators, linear functionals, linear operators and functionals on finite dimensional spaces, normed spaces of operators, Dual spaces. (See Sections 2.2 to 2.10)

UNIT-II

Inner product space, Hilbert space, further properties of inner product spaces, orthogonal complements and direct sums, orthonormal sets and sequences, series related to orthonormal sequences and sets. (Sections 3.1 to 3.5)

UNIT-III

Total Orthonormal sets and sequences, Representation of functionals on Hilbert Spaces, Hilbert-adjoint operator, self-adjoint, unitary and normal operators. (See Sections 3.6, 3.8, 3.9 and 3.10)

UNIT-IV

Hahn-Banach theorems for Complex vector spaces and normed spaces, adjoint operator, Reflexive spaces, uniform boundedness theorem, convergence of sequences of operators and Functionals. Open mapping theorem, closed graph theorem. (See Sections 4.3, 4.5, 4.6, 4.7, 4.12 and 4.13).

Text Book:-Introductory Functional Analysis by E.Kreyszig, John-wiley and Sons, New York, 1978.

References Books:-

- 1).B.V.Limaye, "Functional Analysis", 2nd Edition
- 2). Brown and Page, "Elements of Functional Analysis"
- 3).P.K.Jain, O.P.Ahuja and Khalil Ahmed, "Functional Analysis".

MM 204 Paper-IV

Theory of Ordinary Differential Equations

UNIT-I

Linear differential equations of higher order: Introduction-Higher order equations —A Modelling-Linear independence- Equations with constant coefficients- Equations with variable coefficients- Wronskian- Variation of parameters- Some standard methods.

UNIT-II

Systems of linear differential equations: Introduction – systems of first order equations – existence and uniqueness theorem – fundamental matrix – nonhomogeneous linear systems – linear systems with constant co-efficients – linear systems with periodic coefficients

UNIT-III

Existence and Uniqueness of solutions: Introduction – preliminaries – successive approximations – Picard's theorem – continuation and dependence on initial conditions – existence and solutions on the large – existence and uniqueness of solutions of systems – fixed point method.

UNIT-IV

Analysis and methods of non-linear differential equations:-Introduction – Existence theorem- External solutions – Upper and Lower solutions- Monotone iterative method- Bihari's inequality.

Text Book:

1) Ordinary Differential Equations and Stability Theory by S.G. Deo, V. Lakshmikantham, V. Raghavendra

Reference Books:

- 1. An Introduction to Ordinary Differential Equations by E. A. Coddington
- 2. Ordinary Differential Equations and Stability Theory by David Sanchez
- 3. An Introduction to the Theory of Ordinary Differential Equations by Walter Leighton

MM 205 Paper-V

Discrete Mathematics

UNIT-I

LATTICES: Partial Ordering – Lattices as Posets – some properties of Lattices – Lattices as Algebraic Systems – Sublattices, Direct products and Homomorphisms – some special Lattices – Complete, complemented and distributive lattices. (Pages 183-192, 378-397 of [1])

UNIT-II

BOOLEAN ALGEBRA: Boolean Algebras as Lattices – Bolean Identities – the switching Algebra – sub algebra, Direct product and homomorphism – Join irreducible elements – Atoms (minterms) – Boolean forms and their equivalence – minterm Boolean forms – Sum of products canonical forms – values of Boolean expressions and Boolean functions – Minimization of Boolean functions – the Karnaugh map method.

(Pages 397 - 436 of [1])

UNIT-III

GRAPHS AND PLANAR GRAPHS: Directed and undirected graphs – Isomorphism of graphs – subgraph – complete graph – multigraphs and weighted graphs – paths – simple and elementary paths – circuits – connectedness – shortest paths in weighted graphs – Eulerian paths and circuits – Incoming degree and outgoing degree of a vertex - Hamiltonian paths and circuits – Planar graphs – Euler's formula for planar graphs.

(Pages 137-159, 168-186 of [2])

UNIT-IV

TREES AND CUT-SETS: Properties of trees – Equivalent definitions of trees – Rooted trees – Binary trees – path lengths in rooted trees – Prefix codes – Binary search trees – Spanning trees and Cut-sets – Minimum spanning trees (Pages 187-213 of [2])

Text Books:-

- [1] J P Tremblay and R. Manohar: Discrete Mathematical Structures with applications to Computer Science, McGraw Hill Book Company
- [2] C L Liu: Elements of Discrete Mathematics, Tata McGraw Hill Publishing Company Ltd. New Delhi. (Second Edition).

MM 301 Paper I

Complex Analysis

Unit I

The spherical representation of Complex numbers – Limits and Continuity – Analytic Functions – Polynomials – Rational functions – Sequences – Series – Uniform Convergence – Power series – Abel's Limit Theorem – The Exponential – The Trigonometric Functions. The periodicity – The Logarithm.

Unit II

Def. of Region – Arcs and Closed Curves – Analytic functions in regions – Conformal mapping – Length and area – The Linear Group – The cross ratio – Symmetry – Oriented Circles – Families of Circles.

Unit III

Line Integrals – Rectifiable arcs – Line integrals as functions of arcs – Cauchy's theorem for a rectangle – Cauchy's theorem in a disk – The index of a point w.r.t. a closed curve - The Integral formula – Higher derivatives.

Unit IV

Removable singularities – Taylor's theorem – Zeroes and poles – Local mapping – The maximum principle – Chains and cycles - Simple connectivity – Homology – The general statement of Cauchy's theorem – Proof of Cauchy's theorem.

Text Book:

[1] Lars V. Ahlfors, Complex Analysis, International Student Edition, Third Edition.

Reference Books:

- [2] John B. Conway, Functions of One Complex Variable, Springer International Edition, Narosa Publishing House, 1989
- [3] S. Ponnu Swamy, Foundations of Complex Analysis, Narosa Publishing House, 1995.

MM 302 Paper II

Elementary Operator Theory

Unit I

Spectral theory in finite dimensional normed spaces - Basic concepts of spectrum - Resolvent sets - Spectral properties of bounded linear operators - Further properties of resolvent and spectrum. (Sections 7.1, 7.2, 7.3 and 7.4 of [1])

Unit II

Compact linear operators on normed spaces - Properties of compact linear operators - Spectral properties of compact linear operators on normed spaces - Operator equations involving compact linear operators. (Sections 8.1, 8.2, 8.3 and 8.5 of [1])

Unit III

Spectral properties of bounded self adjoint linear operators - Further spectral properties of bounded linear operators - Positive operators - Square root of a positive operator. (Sections 9.1, 9.2, 9.3 and 9.4 of [1])

Unit IV

Projection operators - Properties of projection operators - Spectral family - Spectral family of a bounded self adjoint linear operator. (Sections 9.5, 9.6, 9.7 and 9.8 of [1])

Text Book:

[1] E.Kreyszig: Introductory Functional Analysis, John Wiley and Sons, New York, 1978.

Reference Books:

- [1] Brown and Page: Elements of Functional Analysis, D.V.N. Comp.
- [2] B.V. Limaye: Functional Analysis, Wiley Eastern Limited, (2nd Edition)
- [3] P.R.Halmos: A Hilbert Space Problem Book, D.Van Nostrand Company, Inc. 1967.

MM 303 A Paper III

Mechanics

Unit I

Dynamics of systems of Particles:- Introduction - Centre of Mass and Linear Momentum of a system- Angular momentum and Kinetic Energy of a system, Mechanics of Rigid bodies- Planar motion:- Centre of mass of Rigid body-some theorem of Static equilibrium of a Rigid body- Equilibrium in a uniform gravitational field- Rotation of a Rigid body about a fixed axis.

Unit II

Moment of Inertia:- calculation of moment of Inertia Perpendicular and Parallel axis theorem- Physical pendulum-A general theorem concerning Angular momentum-Laminar Motion of a Rigid body-Body rolling down an inclined plane (with and without slipping).

Unit III

Motion of Rigid bodies in three dimension-Angular momentum of Rigid body products of Inertia, Principles axes-Determination of principles axes- Rotational Kinetic Energy of Rigid body- Momentum of Inertia of a Rigid body about an arbitrary axis- The momental ellipsoid - Euler's equation of motion of a Rigid body.

Unit IV

Lagrange Mechanics:-Generalized Coordinates-Generalized forces-Lagrange's Equations and their applications-Generalized momentum-Ignorable coordinates-Hamilton's variational principle-Hamilton function-Hamilton's Equations-Problems-Theorems.

Text Book:

[1] G.R.Fowles, Analytical Mechanics, CBS Publishing, 1986.

MM 303 B Paper III

Analytical Number Theory

Unit I

Averages of arithmetical functions: Introduction, The Big Oh notation, Asymptotic equality of functions, Euler's summation formula, Some elementary asymptotic formulas, the average order of d(n), the average order of divisor functions of (n), the average order of (n), an application to the distribution of lattice points, visible from origin, the average order of (n) and (n), the partial sums of a Dirichlet product, application to (n) and (n) another identity for the partial sums of a Dirichlet product. (See 3.1 to 3.12)

Unit II

Some elementary theorems on the distribution of Prime numbers – Introduction, Chebyshev's functions, (n) and (n), Relation connecting (n) and (n) some equivalent forms of the prime number inequalities for (n) and P_n . (See 4.1 to 4.5)

Unit III

Shapiro's Tanberain theorem, applications of Shapiro's theorem, Asymptotic formula for the partial sum, 1/p, The partial sums of the Mobius function, Selberg asymptotic formula. (See 4.6 to 4.11 except 4.10).

Unit IV

Finite abelian groups and their characters, Construction of subgroups, Characters of finite abelian groups, The character group. The orthogonality relations for characters, Dirichlet characters, Sums involving Dirichlet characters, The non-vanishing of L(1,) for real non principal. Dirichlet theorem on primes in Arithmetic progression: Introduction, Dirichlet theorem for primes of the form 4n-1 and 4n+1, The plan of the proof of Dirichlet theorem, Proof of lemma: proof of lemma 7.4, proof of lemma 7.5, proof of lemma 7.6 and proof of lemma 7.7. Distribution of primes in Arithmetic progression (Chapter 6.4 to 6.10, Chapter 7.1 to 7.9)

Text Book:

[1] Tom M. Apostol - Introduction to Analytical Number Theory.

MM 303 C Paper III

Operations Research

Unit I

Formulation of Linear Programming problems, Graphical solution of Linear Programming problem, General formulation of Linear Programming problems, Standard and Matrix forms of Linear Programming problems, Simplex Method, Two-phase method, Big-M method, Method to resolve degeneracy in Linear Programming problem, Alternative optimal solutions.

Unit II

Solution of simultaneous equations by simplex Method, Inverse of a Matrix by simplex Method, Concept of Duality in Linear Programming, Comparison of solutions of the Dual and its primal, Dual Simplex Method, Sensitivity Analysis, Variations in the components (bj) of vector b, Variations in the components (a_{kj}) of matrix A, Addition of new variables and new constraints.

Unit III

Mathematical formulation of Assignment problem, Reduction theorem, Hungarian Assignment Method, Travelling salesman problem, Formulation of Travelling Salesman problem as an Assignment problem, Solution procedure.

Mathematical formulation of Transportation problem, Tabular representation, Methods to find initial basic feasible solution, North West corner rule, Lowest cost entry method, Vogel's approximation methods, Optimality test, Method of finding optimal solution, Degeneracy in transportation problem, Method to resolve degeneracy, Unbalanced transportation problem.

Unit IV

Concept of Dynamic programming, Bellman's principle of optimality, characteristics of Dynamic programming problem, Backward and Forward recursive approach, Minimum path problem, Single Additive constraint and Multiplicatively separable return, Single Additively constraint and Additively separable return, Single Multiplicatively constraint and Additively separable return.

Text Books:

- [1] S. D. Sharma, Operations Research.
- [2] Kanti Swarup, P. K. Gupta and Manmohan, Operations Research.
- [3] H. A. Taha, Operations Research An Introduction.

MM 304 A Paper IV

Commutative Rings

Unit I

Rings and Ideals - Ring and ring homomorphisms - Ideals - Quotient rings - Zero divisors - Nilpotent elements — Units - Prime ideals and maximal ideals - Nil radical and Jacobson radical - Operation on ideals - Extension and contraction (page 1 to 10).

Unit II

MODULES: Modules and module homomorphisms - Submodules and quotient modules - Operations on sub modules - Direct sum and products - Finitely generated modules - Exact sequences - Tensor product s of modules - Exactness properties of tensor products. (pages 17 - 29)

Unit III

Rings of modules of fractions - Local properties extended and contracted ideals of fractions (pages 36 to 43) – primary Decomposition (pages 50 - 54)

Unit IV

Chain conditions (pages 74 - 77) - Notherian rings (pages 80 to 83) - Artinian rings (pages 89 to 91)

Text Book:

[1] M.F.Atiyah (FRS), I.G. McDonald, Introduction to Commutative Algebra, Addison Wesley Publication Company

Reference Book:

[2] S.Gopala Krishnan, Commutative Algebra, published by Oxonian Press Pvt. Ltd. N-56 Connaught Circus, New Delhi - 110001

MM 304 B Paper IV

Integral Equations

Unit I

Basic concepts - Relationship between Linear differential equations and Volterra Integral equations - Resolvent Kernel of Volterra Integral equation. Differentiation of some resolvent kernels - Solution of Integral equation by resolvent kernel - The method of successive approximations - Convolution type equations.

Unit II

Solution of Integro-differential equations with the aid of the Laplace Transformation - Volterra integral equation of the first kind -VIE of the first kind of the convolution type - Eulers integrals - Beta and Gamma functions and their elementary properties - Relationship between Beta and Gamma functions - Abel's problem - Abel's integral equation and its generalizations.

Unit III

Fredholm integral equations of the second kind - Fundamentals - Iterated kernels constructing the resolvent kernel with the aid of iterated kernels - Integral equations with degenerated kernels - Hammerstein type equation - Characteristic numbers and Eigen functions and their external properties.

Unit IV

Solution of Homogeneous Integral equations with degenerate kernels - Green's function - Construction of Green's function for ordinary differential equations - Special case of Green's function - Using Green's function in the solution of boundary value problem - Hilbert-Schmidt theory for symmetric kernels - Schmidt's solution of the non-homogeneous integral equation - Solution of the FIE of first kind.

Text Book:

[1] M. Krasnov, A. Kiselev, G. Makarenko, Problems and Exercises in Integral Equations

MM 304 C Paper IV

Coding Theory

Unit I

Introductory concepts – The communication channel – Linear Binary Code – Generator Matrix – Parity check matrix – Equivalence of codes – Weight, minimum weight and maximum likely hood decoding.

Unit II

Syndrome decoding – perfect codes, Hamming codes, Sphere packing bound – Packing radius, covering radius, MDS codes and some bounds – Varshamov-Gilbert bound – Self Dual codes – Golay codes – Reed Muller codes - Puncturing, extending and shortening

Unit III

A double error correcting BCH code and a finite field of 16 elements – Finite fields – structure of a finite field – Minimal polynomials – Factoring $x^n - 1$.

Unit IV

Cyclic codes – origin and definition – Generator polynomial of a cyclic code – Generator polynomial of the dual code – Idempotent and minimal ideals for binary cyclic codes.

Text Book:

[1] Vera Pless, Introduction to the Theory of Error-Correcting Codes, Wiley Interscience Publication, John-Wiley and Sons Inc., 1998 (Third Edition)

MM 305 A Paper V

Algebraic Number Theory

Unit I

The Gaussian integers – Introduction – The Fundamental theorem of arithmetic in the Gaussian integers - The two square problems.

Unit II

Arithmetic in quadratic fields – Introduction - Quadratic fields - The integers of a quadratic field - Binary quadratic forms - Modules

Unit III

The coefficient ring of a module - The Unit theorem - Factorization theory in quadratic field - The failure of unique factorization.

Unit IV

Generalized congruences and norm of a module - Product and Sum of modules - The Fundamental factorization theorem.

Text Book:

[1] William W. Adam, Lory Joel Goldstein, Introduction to Number Theory. (Sections: 7.1 to 7.3, 8.1 to 8.7, 9.1 to 9.4)

MM 305 B Paper V

Applications of Functional Analysis

Unit I

Concepts of Metric spaces and vector spaces (Questions should not be framed from this part) - Concepts of Normed and Banach spaces - Inner product and Hilbert spaces - Uniform boundedness theorem - Space of polynomials - Fourier series - Strong and weak convergence - Examples - Convergence of sequences of operators and functionals - Applications to summability of sequences. (See 2.2, 3.1, 4.7, 4.8, 4.9, 4.10 of [1])

Unit II

Banach fixed point theorem - Application of Banach theorem to linear equations - Application of Banach theorem to differential equations - Application of Banach theorem to Integral equations. (See 5.1, 5.2, 5.3 and 5.4 of [1])

Unit III

Approximations in Normed spaces – Examples - Uniqueness - Strict convexity - Uniform Approximation. (See 6.1, 6.2, 6.3 of [1])

Unit IV

Legendre, Hermite and Lagurre's polynomials - Chebyshev polynomials - Approximation in Hilbert space - Splines. (See 3.7, 6.4, 6.5 and 6.6 of [1])

Text Book:

[1] Krayszig, Introductory Functional Analysis with Applications, John Wiley and Sons - 1989.

Reference Book:

[2] Brown and Page, Introductory Functional Analysis.

MM 305 C Paper V

Numerical Techniques

Unit I

Bisection Method - Iteration methods - based on first degree equation - Iteration method based on second degree equation - Rate of convergence - (Algorithms on these methods).

Unit II

System of Linear algebraic equations - Direct method - Gaussian elimination method - Triangularization method - Partition method - Iterative method - Gauss Siedel iterative method.

Unit III

Finite difference operators - Interpolation polynomials using finite differences – Gauss, Weddle's, Everett's interpolation - Hermite interpolation - Least square approximation - Lagrange and Newton interpolation. (Algorithms on Lagrange's, and Newton's interpolations). I'

Unit IV

Numerical differentiation - Methods based on interpolation - Methods based on finite differences - Composite integration methods - Trapezoidal rule - Simpson's rule - Euler's method - Taylor series method - Runge-Kutta methods - Predictor Corrector methods - (Algorithms for the above methods - except Predictor- Corrector methods).

Text Books:

- [1] M. K. Jain, S. R. K. Iyengar and R.K. Jain, Numerical Methods for Scientists and Engineers, New Age Intl. Ltd., New Delhi.
- [2] S.S.Sastry: Introduction to Numerical Analysis, Prentice Hall Publications.

MM 401 Paper I

Advanced Complex Analysis

Unit I

The Residue theorem - The argument principle – Evaluation of definite Integrals.

Unit II

Harmonic functions – Definition and Basic properties – The Mean –Value property – Poisson's formula – Schwarz's theorem – The reflection principle.

Unit III

Power series expansions – Weierstrass theorem – The Taylor's series – The Laurent's series – Partial fractions – Infinite products – Canonical products – The Gamma function – Stirling's formula.

Unit IV

Entire functions - Jensen's formula - Hadamard's theorem - The Riemann zeta function - The product development - Extension of (s) to the whole plane - The functional equation - The zeroes of the zeta function.

Text Book:

[1] Lars V. Ahlfors, Complex Analysis, International Student Edition, Third Edition.

Reference Books:

- [2] John B. Conway, Functions of One Complex Variable, Springer International Edition, Narosa Publishing House, 1989
- [3] S. Ponnu Swamy, Foundations of Complex Analysis, Narosa Publishing House, 1995.

MM 402 Paper II

General Measure Theory

Unit I

Measure spaces - Measurable functions — Integration - General Convergence theorems

Unit II

Signed Measures - The Raydon-Nikodym theorem - The L^{P} Spaces.

Unit III

Outer measure and measurability - The Extension theorem - The Lebesque-Stieltjes integral - Product measures.

Unit IV

Inner measure - Extension by sets of measure zero - Caratheodory outer measure - Hausdorff measure.

Scope as in chapters 6,11,12 of Real Analysis by HoLoRoyden.

Text Book:

[1] H.L.Royden, Real Anlysis, Macmillan Publishing Company Inc.,

MM403 A Paper III

Object Oriented Programming

Unit I

Object - oriented programming - Procedural oriented programming - OOP Terminology - Data Abstraction - Data Encapsulation - Objects, classes - Defining member functions - constructors -dynamic initialization of the objects - polymorphism - Function overloading, operator overloading

Unit II

Introduction to computer programming - Programming Fundamentals - Higher Level Language, Operating Systems, Compiling Programs, writing a program in C++ by usage of variables, Data types, Constants, Arithmetic Expression

Unit III

Programming using Control Structures - looping - for statement-while statement do statement - decision making - if statement - switch statement - conditional expression operator - Arrays-Initializing arrays, character arrays-Multi dimensional arrays

Unit IV

Inheritance: Defining derived classes-single and multiple inheritance, virtual base classes, abstract classes, runtime polymorphism and its implementation, virtual functions, dynamic binding. I/O - Console I/O operator in C++, Streams-stream classes - unformatted I/O operations. Exception handling -Templates-Functional Templates.

Text Book:

[1] E.Balaguruswamy: Introduction to C++, Tata Mc Graw Hill

Reference Book:

[2] Venugopal, Ravishankar and Rajkumar: Mastering C++, Tata McGraw Hill

MM 403 B Paper III

Prime Number Theory

Unit I

Dirichlet theorem as primes in an arithmetic progression - primes of the form 4n-l, 4n+1 - Dirichlet of primes in arithmetic progression - Dirichlet series and Euler products - the half plane of convergence of a Dirichlet series - function defined by a Dirichlet series - Multiplication of Dirichlet series - Euler products - the half plane of convergence of a Dirichlet series.

Unit II

Analytic properties of Dirichlet series - Dirichlet series with non negative coefficients - Dirichlet series expressed as exponentials of Dirichlet series - Mean value simule for Dirichlet series - on integral principle for the coefficients of a Dirichlet series and for the partial sum of a Dirichlet series.

Unit III

The function $\zeta(s)$ and $L(s,\chi)$ - integral representation for the Hurwitz function - a contour integral representation for the Hurwitz zeta function - Analytic continuation of the Hurwitz zeta function - analytic continuation of $\zeta(s)$ and $L(s,\chi)$ - Hurwitz formulae for $\zeta(s,\alpha)$ for functional equation for the Riemann Zeta function and Hurwitz zeta function.

Unit IV

Analytic proof of the prime number theorem - plan of the proof - two lemmas proving $[\psi_1(x)]/x^2$ implies prime number theorem - a contour integral representation for $[\psi_1(x)]/x^2$ - upper bounds for $|\zeta(s)|$ and $|\zeta'(s)|$ near the line σ =1 - the non vanishing of $\zeta(s)$ on the line σ =1 - inequalities for $|1/|\zeta(s)|$ and $||\zeta'(s)|/|\zeta(s)||$ - completion of the proof of prime number theorem. Scope as in Chapters: 7, 11, 12, 13 of [1]

Text Book:

[1] Tom. M. Apostol, Introduction to Analytic Number Theory, Springer International Student Edition.

MM 403C Paper III

Advanced Operations Research

Unit I

Characteristics of Game theory - Minimax (Maxmin) criterion and optimal strategy - Saddle points - Solution of Games with saddle points - Rectangular Games without saddle points - Minimax (Maxmin) principle for Mixed strategy Games - Equivalence of Rectangular Game and Linear Programming problem - Solution of (m x n) Games by Simplex method - Arithmetic method for (2 x2) Games - Concept of Dominance - Graphical method for (2 x n) and (m x 2) Games - Method of subgames - Matrix method for (3 x 3) Games without saddle point.

Unit II

Historical development of CPM/PERT Techniques - Basic steps - Network diagram representation - Rules for drawing networks - Forward pass and Backward pass computations - Determination of floats - Determination of critical path - Project evaluation and review techniques updating.

Unit III

Non-Linear programming - Unconstrained problems of Maxima and Minima - Constrained problems of Maxima and Minima - Constraints in the form of Equations - Lagrangian Method - Sufficient conditions for Max (Min) of Objective function with single equality constraint - With more than one equality constraints - Constraints in the form of inequalities - Formulation of Non-Linear programming problems - General Nonlinear programming problem - Canonical form - Graphical Solution

Unit IV

Quadratic Programming - Kuhn-Tucker Conditions - Non-negative constraints, General Quadratic Programming problem - Wolfe's modified simplex method - Beale's Method - Simplex method for Quadratic Programming.

Text Books:

- [1] S.D. Sharma, Operations Research.
- [2] Kanti Swarup, P.K.Gupta and Manmohan, Operations Research.
- [3] O. L. Mangasarian, Non-Linear Programming, McGraw Hill, New Delhi.

MM404 A Paper IV

Banach Algebra

Unit I

Definition of Banach Algebra and examples- Invertib1lity in a Banach Algebra with unity- singular and non-singular elements- Resolvent and spectrum of an element- the spectral radius- Gelfand formula.

(Sections 49, 50, 51 and 55 of Ch.6)

Unit II

Multiplicative linear functionals and the maximal ideal space- the Gelfand Transforms (i.e.:section:Gelfand representation theorem)- the spectral mapping theorem- isometric Gelfand Transform- Topological divisors of zero- boundary of the spectrum- spectrum in L (E).

(Sections: 52, 53, 56 and 57 of Ch. 6)

Unit III

Definition and examples of C-algebra - Self adjoint, unitary, normal, positive elements in C – homomorphisms - representation of commutative algebras, states on C-algebras.

(Sections: 58, 59, 60 and 61 of Ch. 7)

Unit IV

Gelfand Newmark representation theorem - the spectral theorem - the continuous functional calculus - spectral sets.

(Sections: 62 of Ch. 7 and 65, 66 of Ch. 8)

Text Book:

[1] S.K.Berberian, Lectures in Functional Analysis and Operator Theory, Springer International student Ed.

Reference Books:

- [2] Keue Zhu, An Introduction to Operator Algebras, CRC Press, 1963.
- [3] T.W.Palmea, Banach Algebra, Vol- 1, Cambridge University Press, 1994.

MM 404 B Paper IV

Finite Projective Planes

Unit I

Elementary Results:

1.Projective planes2. Subplanes3.Residues Structures4.Projective planes5.Dualty6.The principles of duality

7. Desargue's configuration (Pages 1 to 14 of book [1])

Unit II

Finite Planes:

Counting Lemmas
 Collineations
 Collineations
 Collineations
 Combinatorial results
 Combinatorial results

Field Planes:

1.Fields 2.Prime fields 3.Field planes

4. Matrices and Collineations of $PG(z, p_n)$

5. Analytic Geometry - Coordinates.

(Pages 27 to 42 of [1])

Unit III

Coordinates in an arbitrary Planes:

1. Naming points and lines.

2. The planar ternary rings

3. Properties of (R, F) 4. Collineations and ternary rings (Pages 43 to 53 of [1])

Unit IV

Central Collineations and the Little Desargue's Property

1. Central Collineations 2. Little Desargues property

3. Coordinatization theorems

The Fundamental theorem:

1. Coordinates in a field plane 2, Wedderbern's theorem

3. The fundamental theorem 4. Pappus property.

(Pages 54 to 82 of [1])

Text Book:

[1] A.A. Albert, R. Sandler, An Introduction to Finite Projective Planes

Reference Book:

[2] Hughes and Piper, Projective Planes, Springer Verlag

MM 404 C Paper IV

Fluid Mechanics

Unit I

General orthogonal curvilinear coordinates - Kinematics - Lagrangian and Eulerian methods - Equation of continuity - Boundary surface - Stream lines, Path lines and Streak lines - Velocity potential - Irrotational and rotational motions - Vortex lines

Unit II

Equation of motion - Lagrange's and Euler's equation of motion - Bernoulli's theorem - Stream functions - Irrotational motion in two-dimensions - Complex velocity potential sources - Sinks, doublets and their images - Milne-Thompson Circle theorem

Unit III

Two dimensional irrotational motion produced by motion of Circular, Co-axial and elliptic cylinders in an infinite mass of liquid - Theorem of Blasius motion of a sphere through a liquid at rest at infinity - Liquid streaming past a fixed sphere.

Unit IV

Stress components in a real fluid - Relation between rectangular components of stress - Connection between stresses and gradient of velocity - Navier-Stoke's equations of motion - Plane Poiseulle and couette flows between two parallel plates.

Text Books:

- [1] W.H. Besaint and A.S.Ramsay, A Treatise on Hydromechanics, Part-II. CBS Publishers, Delhi, 1988.
- [2] F.Chorlton, Text book of Fluid Dynamics, CBS Publishers, Delhi, 1985.

MM 405 A Paper V

Calculus of Variations

Unit I

Definitions of Functionals - Strong and Weak Variations - Derivations of Euler's' equation - Other forms of Euler's equation - Special cases - Examples - Fundamental Lemma of Calculus of Variation

Unit II

The problem of minimum surface of revolution - Minimum Energy Problem Brachistochrone Problem - Variational notation - Variational problems involving Several functions.

Unit III

Isoperimetric problem - Examples - Eulers's equations in two dependent variables variational problems in parametric form - Functional dependent on higher order derivatives.

Unit IV

Euler Poisson equation - Natural boundary conditions - Application of Calculus of Variation - Hamilton's principle - Lagrange's Equation.

Text Book:

[1] L. Elsgolts, Differential Equation and Calculus of Variations.

MM 405 B Paper V

Number Theory and Cryptography

Unit I

Some topics in Elementary Number Theory - Time estimates for doing arithmetic - Divisibility and the Euclidean algorithm - Congruences - Some applications to factoring. (Chapter I of [1])

Unit II

Finite fields and Quadratic reciprocity: Finite fields – Quadratic residues - Quadratic reciprocity Law. (Chapter II of [1]).

Unit III

Cryptography: Some simple cryptosystems - Enciphering matrices. (Chapter III of [1])

Unit IV

Public Key - The idea of public key cryptography - RSA - Discrete log - Knapsack - Zero knowledge protocols and oblivious transfer. (Chapter IV of [1])

Text Book:

[1] Neal Koblitz: A Course in Number Theory and Cryptography, Springer, Second Edition, 1994.

MM 405 C Paper V

Dynamical Systems

Unit I

Linear Differential Equations - the adjoint equation - self adjoint equation of second order - the Riccati equation – Green's function.

Unit II

Self adjoint linear differential equations of second order – Abel's formula - The number of zeros on a finite interval - The Sturm separation theorem - The Sturm comparison theorem - The Sturm Picone theorem - The Bocher Osgood theorem - A special pair of solutions - Oscillation on half axis

Unit III

Stability of linear and nonlinear systems - Elementary critical points - Systems of equations with constant coefficients - Linear equations with constant coefficients - Lyapunov stability.

Unit IV

Stability of quasi-linear systems - Second order linear differential equations - Equations with deviating arguments - Equations with constant delay - Equations with piecewise constant delay - A few other types of delay equations.

Text Books:

- [1] Walter Leighton, An Introduction To The Theory Of Ordinary Differential Equations, Wadsworth Pub. Company, Inc. Belmont, California, 1976
- [2] S.G.Deo, V. Lakshmikantham and V. Raghavendra, Text Book of Ordinary Differential Equations (Second Edition)

Reference Books:

- [3] David Sanchez, Ordinary Differential Equations and Stability Theory An Introduction.
- [4] C.A.Swanson, Comparison and Oscillation Theory of Linear Differential Equations, Academic, New York, 1968

MM 451

Practicals: Object Oriented Programming through C++

- 1. Write a programme to find the GCD of two given integers.
- 2. Write a programme to generate the first fifty numbers of Fibonacci Sequence.
- 3. Write a programme for finding the Sum of two matrices A_{mxn} and B_{mxn} .
- 4. Write a function sub-programme to find the transpose of a given matrix A_{mxn} and call it in main programme.
- 5. Write a Programme for finding the product of two matrices A_{mxn} and B_{nxm} ,
- 6. Write a programme for finding the root of an equation using Regular- Falsi method.
- 7. Write a programme for finding the root of an equation using Newton Raphson method
- 8. Write a programme for implementing Gauss Elimination method
- 9. Write a programme to implement Trapezoidal Rule.
- 10. Write a programme to implement Simpsons 1/3 Rule
- 11. Write a programme to implement modified Euler's method.
- 12. Write a programme to implement Runge Kutta method.